Synthetic Evaluation of Power Quality Based on Prospect Theory and Membership Grade

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Abstract: In view of the problem of synthetic evaluation of power quality, considering that measured data of observation points vary during monitoring period, their values may be clear numbers, interval numbers and linguistic assessment terms. At the same time, considering that the decision makers have certain expectations for the power quality index, a hybrid multi-attribute synthetic evaluation method of power quality based on prospect theory and membership grade is proposed. Firstly, Decision matrix with clear numbers, interval numbers and linguistic assessment terms is transformed into a prospect decision matrix according to the expectation of each attribute. Then, variable fuzzy pattern recognition model is established according to the generalized weighted Euclidean distance between each alternative and decision expectation. Thirdly, optimal membership grade and corresponding attribute weight of each alternative are obtained by means of building a Lagrange relaxation function to carry out the cross iterative calculation. Finally, by synthesizing accumulative prospect value and membership grade, ranking results of each alternative is determined according to comprehensive prospect value. Synthetic evaluation of power quality is realized.

Keywords: Power quality, Prospect theory, Membership grade, Multi-attribute decision making.

INTRODUCTION

Synthetic evaluation of power quality is an important means to quantify level of power quality. It is the process of evaluating power quality characteristics and checking and judging whether they meet standard requirements based on actual measurement of electrical operating parameters of power system or basic data obtained by modeling and simulation [Lin, et. al., 2013]. Traditional methods usually treat measured data of power quality monitoring points as the clear numbers, but measured data of monitoring points in a monitoring period are often changed. Measured data may be the clear numbers or the interval numbers. Even value of some indexes may be linguistic assessment terms. Synthetic evaluation of power quality is essentially an uncertain multi-attribute decision making problem [JIN, et. al., 2016]. Synthetic evaluation of power quality is a decision-making process based on limited rational behavior of decision makers. Under uncertain conditions, decision makers value benefits and losses relative to a certain reference point rather than final total value. Therefore, using prospect theory to deal with synthetic evaluation of power quality has some advantages.

Currently synthetic evaluation methods of power quality mainly include fuzzy mathematics theory [Jia, et. al., 2000], probability and statistics theory [Huang, et. al., 2003], clustering theory [JIANG, et. al., 2012], catastrophe decision theory [Zeng, et. al., 2003], D-S evidence theory [Chen, et. al., 2012], neural network [Zhou, et. al., 2007] and so on. In reference [Salarvand, et. al., 2010], a two-level evaluation method was proposed to determine the membership function and membership grade of power quality indexes and final result was obtained by fuzzy comprehensive evaluation method. In reference, a method of power quality quantization and evaluation based on daily period was proposed. Main characteristics of power quality were described by using probability statistical eigenvalue and comprehensive quantitative evaluation index of power quality was obtained. In reference, using fuzzy clustering analysis, known data points of power quality grade were added to sample data set for cluster analysis, and power quality grade of data points to be evaluated was determined according to the principle of "similar clustering". In reference, synthetic evaluation method of power quality based on catastrophe decision theory was proposed. Catastrophe model was used to calculate the abrupt progression at all levels, which avoided power indexes weighting and reduces subjectivity of decision making. In reference, D-S evidence theory was used to improve accuracy of each element of judgment matrix and to reflect the comprehensiveness of evaluation object, thus improving accuracy of evaluation results. In reference [Liu, et. al., 2007], evaluation of power quality based on artificial neural network did not need artificial weighting and avoided the influence of subjective factors. However, the reliability of evaluation results was reduced because of inability to investigate weight of power quality indexes.

Based on the above researches, in the current synthetic evaluation of power quality, there are lack of researches on uncertain multi-attribute decision making whose attribute values are clear numbers, interval numbers and linguistic assessment terms and
attribute weights are unknown. A hybrid power quality multi-attribute decision making method based on prospect theory\cite{14-16} and membership grade is proposed in the paper.

**PROSPECT THEORY**

**Hybrid data type**

There is a hybrid multi-attribute decision making problem where the scheme set is \( A = \{ A_1, A_2, A_3 \} \), the attribute set is \( B = \{ B_1, B_2, B_3 \} \), the decision matrix is \( C = [ C_{ij} ]_{mn} \), and the decision maker gives the expectation vector about the attribute \( E = \{ E_1, E_2, E_3 \} \) according to the existing information and expectation of the future. Where \( A \) represents the \( i \) alternative, \( B_j \) represents the \( j \) attribute, \( C_{ij} \) represents the value of the \( j \) attribute of the \( i \) alternative, and each attribute is independent of each other.

As a hybrid multi-attribute decision making problem, attributes will include clear numbers, interval numbers and linguistic assessment terms, which are represented as \( C^N \), \( C' \), \( C^l \) in sequence. Obviously, \( C^N \cup C' \cup C^l = C \), where attribute value of \( C^N \) are an exact number, the attribute value of \( C' \) are an interval number, and the attribute value of \( C^l \) is a linguistic assessment terms, as follows:

1. When \( C_{ij} \in C^N \), \( C_{ij} = B_j \), where \( B_j \) is real numerical value and Supposes \( B_j \geq 0 \).
2. When \( C_{ij} \in C' \), \( C_{ij} = [ B^L_j, B^U_j ] \), where \( B^L_j \), \( B^U_j \) are real numerical value and Supposes \( B^L_j \geq B^U_j \geq 0 \).
3. When \( C_{ij} \in C^l \), \( C_{ij} = B_j \), where \( B_j \in S \). Because it is difficult to for \( C^l \) to use numerical measure, \( S \) is a predefined linguistic state set \( S = \{ S_1, \ldots, S_8 \} \), where \( S_j \) represents the \( j \) linguistic assessment terms of \( S \). \( T \) is even and \( S \) contains \( T+1 \) elements. When \( T = 6 \), \( S = \{ S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8 \} \), the corresponding "very poor, poor, medium, medium, good, very good" seven states, recorded as \( S = \{ VP, P, MP, M, MG, G, VG \} \). To facilitate the processing and computation of linguistic assessment terms, converting linguistic assessment terms into the corresponding triangular fuzzy numbers. If \( C_{ij} = S_j \),

\[
C_{ij} = [ B^L_j, B^U_j, B^M_j ] = \begin{cases} 
\max \{(f-1)/T, 0\}, f/T, \min \{(f+1)/T, 1\} 
\end{cases}
\]

**Data normalization processing**

In hybrid multi-attribute decision making problem, the attributes are divided into benefit type \( B_B \) and cost type \( B_C \). The attributes, the better benefit types are. The smaller attribute value is, the better cost types are. In order to eliminate influence of different physical dimension on decision results, it is necessary to normalize expectation vector and decision matrix. When \( M = \{ 1, 2, L, m \} \) and \( N = \{ 1, 2, L, n \} \). Because of the different attribute types, the data needs to be normalized as follows:

1. When \( C_{ij} \in C^N \), \( i \in M \), as \( P'_j = \max \{ \max \{ C_{ij} \}, E_j \} \), \( P'_j = \min \{ \min \{ C_{ij} \}, E_j \} \), normalized calculation formula is as follows:

\[
C''_{ij} = \frac{\left( P_j - C_{ij} \right)}{\left( P'_j - P_j \right)}, \quad j \in B_B \tag{2}
\]

\[
E''_j = \frac{\left( P_j - E_j \right)}{\left( P'_j - P_j \right)}, \quad j \in B_B \tag{3}
\]

2. When \( C_{ij} \in C' \), \( i \in M \), as \( P'_j = \max \{ \max \{ C_{ij} \}, E_j \} \), \( P'_j = \min \{ \min \{ C_{ij} \}, E_j \} \), normalized calculation formula is as follows:

\[
C''_{ij} = \frac{\left( P''_j - C_{ij} \right)}{\left( P'_j - P''_j \right)}, \quad j \in B_B \tag{4}
\]

\[
E''_j = \frac{\left( P''_j - E_j \right)}{\left( P'_j - P''_j \right)}, \quad j \in B_B \tag{5}
\]

3. When \( C_{ij} \in C^l \), \( i \in M \) and \( C_{ij} \) and \( E_j \) are converted into triangular fuzzy numbers according to formula (1). Decision matrix is \( C = [ C''_{ij} ]_{mn} \) and expectation vector is \( E = [ E'_1, E'_2, L, E'_n ] \) after data normalization processing.

**Determining prospect decision matrix**

In prospect theory, decision makers will measure the "gain" or "loss" of each attribute according to reference points. Therefore, it is necessary to compare attribute values and expectation attributes of each alternative, determine size relationship of alternatives between each attribute and reference point, and calculate the Euclidean distance between each attribute and reference point to determine the prospect decision matrix \( V = [ V_{ij} ]_{mn} \). Due to different types of attributes, comparison methods are different. The specific comparison methods are as follows:

1. When \( C_{ij} \in C^N \), directly comparing the size between \( C''_{ij} \) and \( E''_j \).
(2) When $C_j \in C^i$, as $S(E_j) = (C_j^i + C_0^j)/2$, $S(E_j') = (E_j^i + E_0^j)/2$, $K(E_j') = E_j^i - C_j^i$. When $S(C_j) \neq S(E_j)$, if $S(C_j) > S(E_j)$, $C_j^i > E_j$, if $S(C_j) < S(E_j)$, $C_j^i < E_j$.

(3) When $C_j \in C^i$. If $C_j^i \notin E_j'$, $C_j^i > E_j$. If $C_j \notin E_j'$, $C_j < E_j$. To further calculate Euclidean distance $D_0$ between attribute values $C_0^i$ of each scheme and expectation attribute $E_j$, when $i \in M$, the formula is:

$$D_0 = \sqrt{\sum_{j \in C^i} [(C_0^i - E_j)^2 + (C_0^i - E_j')^2]/2}$$

Each scheme is judged "gain" or "loss" relative to expectation attribute according to size of $C_j$ and $E_j$.

Under $i \in M$, when $C_j^i > E_j'$, the $j$ attribute of the $i$ scheme is benefit relative to expectation attribute $E_j$. Taking into account different risk attitudes of decision makers towards gains and losses, a prospect decision matrix is established, in which prospect value $V(C_j)$ of the $j$ attribute of the $i$ scheme is represented. The calculation formula is as follows:

$$V(C_j) = \begin{cases} (D_0)^{-1}, & C_j \geq E_j \\ -\theta \times (D_0)^{-1}, & C_j < E_j \end{cases}$$

Where $\alpha$ and $\beta$ respectively represent bump degree of value function $V(C_j)$ in benefit region and loss region, and $0 < \alpha < 1$, $0 < \beta < 1$. It can be found that decision maker is a concave function in the face of benefit, showing a risk repugnance and a convex function in the face of loss, showing a risk preference. The larger the value of $\alpha$ and $\beta$ are, the more inclined decision makers are to take risks. $\theta$ represents loss avoidance coefficient of decision makers and $\theta > 1$. The larger the value of $\theta$ is, the greater the degree of loss avoidance of decision makers in the face of loss.

**MEMBERSHIP GRADE AND COMPREHENSIVE PROSPECT VALUE**

Membership grade and attribute weights

The comparison of schemes can only be distinguished under the same criterion. The cumulative prospect value of each scheme must come from weight vector of same attribute. Therefore, variable fuzzy pattern recognition model[17][18] is used to determine the membership grade and attribute weight vector of each scheme and decision expectation.

According to calculation method of distance $D_0$ proposed by formula (6), $D_0 \in [0,1]$. When $D_0 \to 0$, the gap between the $j$ attribute of the $i$ scheme and expectation value of the $j$ attribute of decision makers is smaller, and when $D_0 \to 1$, the gap between the $j$ attribute of the $i$ scheme and expectation value of the $j$ attribute of decision makers is greater. So a two-level opposed fuzzy recognition center $S = \left[ s_{hn} \right]_{n=1}^N$ is set up, in which $h = \{1,2\}$. When $h = 1$, $s_{h1} = 0$ represents the optimal attribute set. When $h = 2$, $s_{h2} = 1$ represents the worst attribute set. $U = \left[ u_{hn} \right]_{n=1}^N$ is denoted as a membership grade matrix, in which the membership grade $u_{hn}$ between the $i$ scheme and the fuzzy recognition center is expressed. When $r_i = D_0$, in order to solve optimal membership grade $u_{hn}$, and optimal weight vector $\omega$ of alternative scheme and class center, relative membership grade $u_{hn}$ and attribute weight vector $\omega$ are introduced. If generalized weighted Euclidean distance between alternative scheme and expectation vector is recorded as $f(\omega,u)$, the weighted generalized Euclidean distance between the $i$ scheme and fuzzy recognition center is:

$$f(\omega,u) = \left( \sum_{h=1}^2 u_{hn} \sqrt{\sum_{n=1}^N \left[ \omega_n \times (r_i - s_{hn})^2 \right]} \right)^2$$

Obviously, the smaller $f(\omega,u)$ is, the smaller the difference between the $i$ scheme and expectation target is and the better the recognition of expectation vector is. $F(\omega,u)$ is regarded as the difference between each scheme and all reference points, then

$$F(\omega,u) = \left[ f_1(u,\omega), f_2(u,\omega), L, f_m(u,\omega) \right]$$

Since there is no focus relationship between the
alternatives, optimization model is established as follows:

$$\min Z = F(u, \omega) = \sum_{j=1}^{n} f_j(u, \omega) = \sum_{j=1}^{n} \sum_{k=1}^{m} \sum_{i=1}^{l} \omega_j \cdot \left( r_{ij} - s_{ih} \right)$$

(9)

under certain constraints:

$$\sum_{k=1}^{l} u_{ih} = 1, \quad 0 \leq u_{ih} \leq 1, \quad \sum_{j=1}^{n} \omega_j = 1, \quad 0 \leq \omega_j \leq 1$$

For the optimization problem, Lagrangian relaxation method is established.

$$L(u, \omega; \lambda_u, \lambda_\omega) = \sum_{k=1}^{l} \left[ \sum_{j=1}^{n} \sum_{i=1}^{l} \omega_j \cdot \left( r_{ij} - s_{ih} \right) \right] - \lambda_u \left( \sum_{k=1}^{l} u_{ih} - 1 \right) - \lambda_\omega \left( \sum_{j=1}^{n} \omega_j - 1 \right)$$

(10)

When \( \partial L / \partial u = 0 \), \( \partial L / \partial \omega = 0 \), \( \partial L / \partial \lambda_u = 0 \), then

$$u_{ih} = \left\{ \frac{2}{\sum_{k=1}^{l} \left[ \sum_{j=1}^{n} \sum_{i=1}^{l} \omega_j \cdot \left( r_{ij} - s_{ih} \right) \right]^2} \right\}^{-1}$$

(11)

$$\omega_j = \left\{ \frac{\sum_{i=1}^{l} \sum_{k=1}^{l} \sum_{j=1}^{n} \omega_j \cdot \left( r_{ij} - s_{ih} \right)^2}{\sum_{k=1}^{l} \sum_{j=1}^{n} \sum_{i=1}^{l} \omega_j \cdot \left( r_{ij} - s_{ih} \right)^2} \right\}^{-1}$$

(12)

In order to obtain optimal membership matrix \( u_{ih} \) and weight vector \( \omega_j \), the cyclic iteration method in the variable fuzzy pattern recognition model is used to solve the problem in the paper.

Ranking and optimal selection

In prospect theory, attribute value of each scheme can be judged as "benefit" or "loss" according to prospect decision matrix. If attribute weight adopts weight vector \( \omega^* = \{ \omega_1, \omega_2, L, \omega_m \} \) in variable fuzzy iteration, cumulative prospect value \( P(A) \) of each scheme can be calculated according to cumulative prospect theory[18]. The formula of the cumulative prospect value of the scheme can be calculated:

$$P(A) = \sum_{j=1}^{n} \omega_j \cdot V(c_i), i \in M$$

(13)

In formula (13), the value of \( V(c_i) \) is calculated according to formula (7). If only cumulative prospect value of the scheme is considered, it is obvious that the bigger \( P(A) \) is, the better scheme \( A_i \) is. The scheme can be sequenced according to value of \( P(A) \).

In variable fuzzy pattern recognition model, the optimal membership matrix \( u_{ih} \) after cross-cycle iterative calculation reflects degree of proximity between each scheme and decision expectation. When \( h=1 \), membership grade \( u_{ih} \) of each scheme and decision expectation is represented. Set \( U(A) \) as membership grade between scheme \( A_i \) and decision-makers expectation, then \( U(A) = u_{ih} \). The larger \( U(A) \) is, the higher membership grade of scheme \( A_i \) is. If we only consider membership grade of schemes, we can sort it according to value of \( U(A) \).

Because prospect value and membership grade reflect the relationship between schemes and decision expectation from two angles, decision should be considered synthetically. If cumulative prospect value of scheme \( A_i \) is larger and membership grade is lower, it indicates that prospect value of scheme \( A_i \) is better. But deviation is large from decision expectation, so scheme \( A_i \) is not the best. If cumulative prospect value of scheme \( A_i \) is lower and membership grade is higher, it indicates that it is close to expectation of decision makers. But prospect value is poor, scheme \( A_i \) is not optimal either. If and only if cumulative prospect value of scheme \( A_i \) is higher and membership grade is higher, scheme \( A_i \) is better. Therefore, cumulative prospect value and membership grade of each scheme are considered synthetically in the paper, that is, the comprehensive prospect value is recorded \( S(A) \). Each scheme is sorted and selected.

The calculation formula is as follows:

$$S(A) = \left\{ \begin{array}{ll} P(A) \times U(A), & \text{when } P(A) \geq 0 \\ [P(A) \times [1 - U(A)]], & \text{when } P(A) < 0 \end{array} \right.$$  

(14)

Comprehensive prospect value in formula (14) takes into account influence of accumulative prospect value on positive values and negative values. The bigger comprehensive prospect value \( S(A) \) is, the better scheme \( A_i \) is.

Example application

In this paper, first of all, the seven power quality standards of the State Grid are taken as the basis for the comprehensive evaluation of power quality. The seven evaluation indexes [12] include voltage deviation \( f_1 \), voltage fluctuation \( f_2 \), voltage flicker \( f_3 \), voltage sag \( f_4 \), three-phase unbalance \( f_5 \), voltage harmonic \( f_6 \) and frequency deviation \( f_7 \). These index values belong to the interval numbers. Then reliability index and service index of power supply are considered. Reliability index of power supply is expressed by average service availability index \( f_1 \), which belongs to clear number. The service index [19] mainly considers the content of demand side management, customer satisfaction and so on. It belongs to linguistic assessment term. Decision-
makers put forward expected goal of power quality, which is transformed into the expected vector:

\[ \mathbf{E} = \{[2.80, 4.85], [1.10, 1.15], [0.336, 0.624], [0.206, 0.537], [0.40, 0.84], [1.55, 2.45], [0.060, 0.095], [0.855, 0.945], \} \]

Initial data of measured power quality index are shown in Tab 1.

<table>
<thead>
<tr>
<th>Observ points</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_1 )</td>
<td>3.21,4.28</td>
<td>5.73,6.68</td>
<td>4.35,5.12</td>
<td>5.33,5.87</td>
<td>4.22,4.79</td>
</tr>
<tr>
<td>( f_2 )</td>
<td>1.33,1.56</td>
<td>1.53,1.72</td>
<td>1.82,1.95</td>
<td>1.37,1.46</td>
<td>1.58,1.69</td>
</tr>
<tr>
<td>( f_3 )</td>
<td>0.672,0.763</td>
<td>0.798,0.846</td>
<td>0.631,0.689</td>
<td>0.453,0.528</td>
<td>0.516,0.592</td>
</tr>
<tr>
<td>( f_4 )</td>
<td>0.582,0.616</td>
<td>0.335,0.427</td>
<td>0.516,0.586</td>
<td>0.468,0.502</td>
<td>0.348,0.410</td>
</tr>
<tr>
<td>( f_5 )</td>
<td>0.83,0.92</td>
<td>0.62,0.70</td>
<td>1.55,1.68</td>
<td>0.94,1.10</td>
<td>0.95,1.06</td>
</tr>
<tr>
<td>( f_6 )</td>
<td>2.72,2.93</td>
<td>2.79,3.28</td>
<td>2.67,2.81</td>
<td>2.36,3.58</td>
<td>2.12,2.87</td>
</tr>
<tr>
<td>( f_7 )</td>
<td>0.093,0.098</td>
<td>0.091,0.095</td>
<td>0.097,0.103</td>
<td>0.099,0.112</td>
<td>0.096,0.098</td>
</tr>
<tr>
<td>( f_8 )</td>
<td>0.885</td>
<td>0.862</td>
<td>0.893</td>
<td>0.840</td>
<td>0.864</td>
</tr>
<tr>
<td>( f_9 )</td>
<td>M</td>
<td>MG</td>
<td>G</td>
<td>G</td>
<td>M</td>
</tr>
</tbody>
</table>

Then, based on variable fuzzy recognition model and cross cycle iteration calculation according to formula (12) ~ (13), the optimal membership matrix is obtained.

\[ u_h^* = \begin{bmatrix} 0.6784 & 0.7068 & 0.7626 & 0.6235 & 0.6013 \\ 0.3216 & 0.2932 & 0.2374 & 0.3765 & 0.3987 \end{bmatrix} \]

optimal weight vector is:

\[ \mathbf{\omega} = \{0.1175, 0.1022, 0.1099, 0.1296, 0.1106, 0.1058, 0.1021, 0.1090, 0.1133\} \]

For \( u_h^* \), when \( h = 1 \), membership grade between alternatives and decision makers is expressed. Membership grade of each scheme is in turn

\[ U = \{0.6784, 0.7068, 0.7626, 0.6235, 0.6013\} \]

According to the formula (13), the cumulative prospect values of each scheme are calculated in order

\[ P = \{0.2221, 0.4308, 0.3175, 0.2460, 0.1010\} \]

Finally, according to formula (14), comprehensive prospect value based on cumulative prospect value and membership grade of each scheme is calculated as

\[ S = \{0.1507, 0.3045, 0.2421, 0.1533, 0.0607\} \]

Scheme ranking results based on membership grade, cumulative prospect value and comprehensive prospect value are shown in Tab 5. It can be seen that the optimal scheme is \( A_1 \), if sorted according to membership grade. The optimal scheme is \( A_2 \), if sorted according to cumulative prospect value. The optimal scheme is \( A_3 \), if sorted according to comprehensive prospect value. The results show that expectation of scheme \( A_3 \) is the most close to that of decision-makers, but prospect value of scheme \( A_1 \) is worse than that of scheme \( A_2 \). When considered synthetically, the final optimal scheme is \( A_2 \).

### CONCLUSIONS

In this paper, evaluation indexes of power quality has the characteristics of clear numbers, interval numbers and linguistic assessment terms in a certain monitoring period. At the same time, decision makers have certain expectation for index values, but also take reference points, decision matrix of profits and losses relative to the reference point is established according to prospect theory, and the fuzzy pattern recognition model is established according to the generalized weighted Euclidean distance between the alternative scheme and the decision-maker undefined expectation. The optimal membership degree and the optimal attribute weight of each scheme and expected target are obtained. Finally, considering the foreground value and membership degree of each option under the expectation of the decision maker, the alternatives are sorted. This method has strong operability and practicability, and puts forward a new idea to solve the problem of comprehensive evaluation of power quality. The method not only deals with various data types in the process of comprehensive evaluation of power quality but also takes expectation of the decision-makers into consideration and incorporate into the model. It has strong operability and practicability. A new idea is put forward to solve the problem of comprehensive evaluation of power quality.
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